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LETTER TO THE EDITOR

The quenching of the quantised Hall resistance at low magnetic fields

R Johnston† and L Schweitzer‡

† GEC Hirst Research Centre, Wembley, Middlesex HA9 7PP, UK

‡ Physikalisch-Technische Bundesanstalt, Bundesallee 100, 33 Braunschweig, Federal Republic of Germany

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Abstract. The quenching of Hall resistance at low magnetic fields is considered theoretically and is shown to be contained in a theory proposed recently to explain the quantum Hall effect without invoking localisation. In this model the quenching of the Hall voltage arises from the mixing of edge states. The Hall resistance of a two-dimensional system, which is confined by a parabolic potential well, is calculated and the results compare well with observations on electrostatically confined samples reported recently by Ford and co-workers.

The recent observation of the complete quenching of the Hall voltage at low magnetic fields in narrow two-dimensional conductors by Roukes and co-workers [1] seems not to have received sufficient attention. Although experimentally reproduced in electrostatically confined systems by Ford and co-workers [2], and theoretically addressed in two Letters [3, 4], the main message of the experiments, namely the breakdown of the commonly accepted theories of the quantum Hall effect (QHE), has apparently not been noticed.

Of the approaches taken using the localisation model, both the linear response theory of the QHE [5, 6], and the edge state descriptions [7–9], which do not really treat the assumed disorder, are in principle not able to account for the new experimental observations [1, 2].

Since in the experiments on samples that show the QHE the Hall voltage vanishes at low magnetic fields ($B < 0.2$ T) without simultaneous vanishing of the current through the sample, the above-mentioned theories turn out to be inapplicable. If for instance a zero 'applied' Hall field is taken as the perturbation in the Kubo formula [5, 6] the current will also vanish. The same happens if the *ad hoc* assumed chemical potential difference is taken to be zero [7–9]. This clearly shows that the commonly accepted method of calculating the Hall current as a response to an externally applied Hall voltage is untenable and a completely different approach to the problem has to be taken.

In this Letter the low-field quenching of the quantised Hall resistance is shown to be contained already in a theory, which has been proposed recently [10] to explain the QHE without assuming Anderson localisation. In contrast to what is done in the usual approaches, the Hall voltage is not treated as an input quantity but the current I through and the Hall voltage U_H across the system are calculated within the theory as a response to an applied electromotive force (battery) and the perpendicular magnetic field $\boldsymbol{B}$. This view is in accordance with the experimental situation, where both the current and the Hall voltage are measured to give the Hall resistance $R_H = U_H/I$.

In this new theoretical description [10], an electric field pulse along the sample disturbs the equilibrium state of the two-dimensional electron gas. The non-equilibrium density matrix is derived by linearising the von Neumann equation in the electric pulse perturbation. The resultant density matrix is used to calculate the current and the Hall voltage, from which the Hall resistance is obtained. R_H can be expressed in terms of the eigenvalues $E_n(k)$ and eigenstates $|n, k\rangle$ of the unperturbed system. In the limit $T \rightarrow 0$ equation (4.16) of [10] gives the Hall resistance:

$$R_H = \frac{h}{e^2}\left[2\sum_{n,i}\int_W \mathrm{d}y\, \langle n, k_i|y\rangle^2 \operatorname{sign}\left(\frac{\partial E_n}{\partial k}\bigg|_{k=k_{i,n}}\right)\right] \times \left(\sum_{n,i}\int_W \mathrm{d}y\, \langle n, k_i|y\rangle^2 \left|\frac{\partial E_n}{\partial k}\right|^{-1}_{k=k_{i,n}} \sum_{n,i}\left|\frac{\partial E_n}{\partial k}\right|_{k=k_{i,n}}\right)^{-1}. \tag{1}$$

The $\{k_{i,n}\}$ are the set of solutions of $E_n(k) = E_F$, where E_F is the Fermi energy, and W is the contact region, where the chemical potential is probed. It is now possible to calculate the Hall resistance for a particular system, provided the eigenvalues and eigenfunctions are given. In this Letter the simple parabolic well model is chosen for which the energy spectrum and the wavefunctions are known analytically [11, 12] in order to demonstrate the quenching of the Hall effect. It has been shown [13, 14] that a square well potential is better suited for modelling broader systems, and it is also possible to express the eigenvalues and eigenfunctions as power series in terms of the system parameters, where the expansion coefficients have to be calculated numerically [15]. For the present purposes, however, the choice of the parabolic well potential, which may be more appropriate in the case of a narrow electrostatic confinement [2], will suffice to demonstrate the low-field quenching but at the sacrifice of exact quantisation of R_H which is expected for broader systems, where a square well potential would be needed.

The following one-particle Hamiltonian describes the motion of non-interacting electrons in a two-dimensional system with confining parabolic well potential in the y direction centred at $y = 0$ and a perpendicular magnetic field $\boldsymbol{B}$ [11, 12]:

$$H = (\hbar^2/2m)\,[-(\omega_0^2/\omega^2)\,(\partial^2/\partial x^2) - (\partial^2/\partial y^2)] + (m/2)\,\omega_0^2(y - Y_0)^2 \tag{2}$$

where $\omega^2 = \omega_0^2 + \omega_c^2$, $\omega_c = eB/m$ and $Y_0 = (p_x/eB)\omega_c^2/\omega^2$. Applying periodic boundary conditions in the x direction, the solution of the Schrödinger equation $H|n, k\rangle = E_n(k)|n, k\rangle$ gives the eigenvalues

$$E_n(k) = \hbar\omega(n - \tfrac{1}{2}) + (\hbar^2/2m)k^2(\omega_0^2/\omega^2) \tag{3}$$

with $n = 1, 2, \ldots,$ and eigenfunctions

$$\langle x, y|n, k\rangle = \mathrm{e}^{ikx}(m\omega/\pi\hbar L^2)^{1/4}[2^{n-1}(n-1)!]^{-1/2}\,\mathrm{e}^{-z^2/2}\,H_{n-1}(z) \tag{4}$$

where $\{H_n\}$ are the Hermite polynomials, and $z = (m\omega/\hbar)^{1/2}(y - Y_0)$.

Inserting the spectrum and eigenfunctions into (1), and performing the integral over k, gives the following expression for the Hall resistance of the parabolic well model

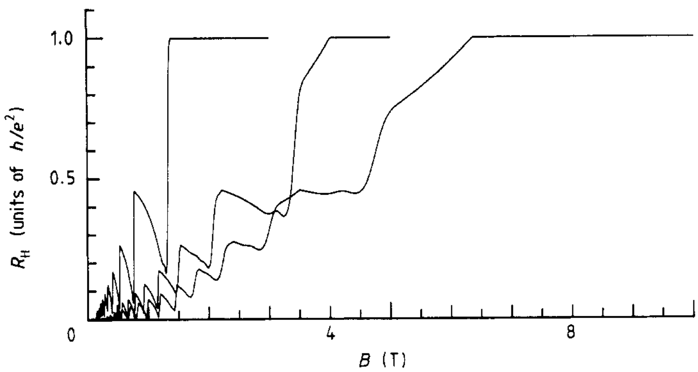

Figure 1. The Hall resistance at $T = 0$ K as a function of magnetic field B for three values of the potential parameter $\omega_0 = 1\times10^{10}, 5\times10^{10}, 1\times10^{11}\ \mathrm{s}^{-1}$. The electron line density is $\rho = 5\times10^{10}\ \mathrm{m}^{-1}$, and $\lambda = 8\times10^{-8}$ m.

considered:

$$R_{\mathrm{H}} = \frac{h}{e^2}\left(2\sum_n \int_{z_n^+}^{z_n^-} \mathrm{d}z\, \mathrm{e}^{-z^2} H_{n-1}^2(z)\right)\left[\sum_n \left(2\int_{-\infty}^{z_n^+} \mathrm{d}z\, \mathrm{e}^{-z^2} H_{n-1}^2(z) + \int_{z_n^+}^{z_n^-} \mathrm{d}z\, \mathrm{e}^{-z^2} H_{n-1}^2(z)\right)|k_n|^{-1}\sum_n |k_n|\right]^{-1} \tag{5}$$

where the symmetry of the system with respect to $y = 0$ was taken into account, and the $\{k_n\}$ are given by

$$k_n = \{(2m/\hbar^2)(\omega^2/\omega_0^2)[E_{\mathrm{F}} - \hbar\omega(n - \tfrac{1}{2})]\}^{1/2} \tag{6}$$

and $z_n^{\pm} = \sqrt{m\omega/\hbar}[-\lambda \mp (\hbar k_n/eB)\omega_c^2/\omega^2]$. Since in the parabolic well model the system has no sharp physical edges, the integral over y, which corresponds to the contact region W where the Hall voltage is probed, was taken between the limits $-\infty$ and $-\lambda$. The results, however, do not essentailly depend on the lower limit as long as it remains outside the effective system width.

In figure 1 the Hall resistance R_{H} is shown as a function of the magnetic field B, for three different values of the potential parameter. The striking resemblance to the results of Ford and co-workers [2] for the strongly electrostatically confined samples is obvious. The low-field quenching of the Hall resistance is clearly observed as well as the non-linear increase of the curves with the magnetic field strength. Notice that, due to the neglect of the spin splitting in the model, the plateau value of R_{H} deviates from the quantised value already for $i = 2$. A second difference comes fom the finite temperatures in the experiments, so the low-field oscillations are not always resolved. In figure 2 R_{H} is shown for $\omega_0 = 1\times10^{10}\ \mathrm{s}^{-1}$ but the particle density ρ is smaller as compared with the first curve in figure 1. It is not clear how ω_0 and ρ are changed simultaneoulsy in the experiments by changing the gate voltage. The inset of figure 2 shows the low-field quenching in detail. The conjecture of Roukes and co-workers [1], that the Hall resistance assumes a last plateau, from which it drops to zero when lowering the magnetic field, is not corroborated by the present investigation. As can be seen from the inset of figure 2, the quenching has already started at band level index $n = 13$ although there are 23 subbands occupied in the limit $B \to 0$.

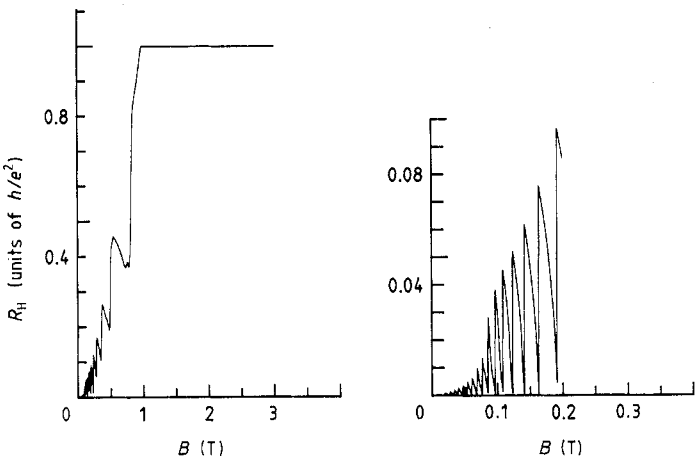

Figure 2. The Hall resistance at $T = 0$ K versus B for $\omega_0 = 1 \times 10^{10}\ \mathrm{s}^{-1}$, $\rho = 3 \times 10^8\ \mathrm{m}^{-1}$ and $\lambda = 2 \times 10^{-7}$ m. The inset shows the low-field behaviour in detail. The quenching of R_H starts at band index $n = 13$ and tends to zero for the last index $n = 23$.

In this theory the mechanism that is responsible for quenching the Hall voltage at low magnetic fields is edge state mixing. Edge states are present in the parabolic potential model as well as in the square well case. For this reason it is expected that the Hall voltage will be quenched in the square well model too, but the dependence of the Hall resistance on the magnetic field remains to be calculated. It is not clear how much, if any, of the structure shown in figure 2 remains in the square well model. As the model with a square potential well is expected to be of more relevance to the experiments on narrow wires by Roukes and co-workers [1], it is hoped that the calculation can be performed in the near future. It is proposed that strong indications of the detailed structure shown in figure 2 have already been observed by Ford and co-workers [2] in experiments on electrostatically confined systems.

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